1. INTRODUCTION TO SHAFT SENSORLESS FORCED DYNAMICS CONTROL OF AC ELECTRICAL DRIVES

1.1. Basic Principles

1.1.1 Forced dynamic modes

The new forced dynamic control law for electric drives employing AC motors enables various prescribed dynamic responses to speed demands to be obtained, which will be referred to as dynamic modes. Those considered here are:

a) the *direct acceleration control* mode, in which the rotor acceleration follows a time-varying acceleration demand with virtually zero dynamic lag,

b) the *constant acceleration* mode, in which the rotor speed is automatically controlled to respond to piecewise constant reference speed changes with a prescribed constant acceleration (or deceleration),

c) the *constant jerk* (rate of change of acceleration) mode, in which the rotor acceleration during speed-up is controlled to linearly increase up to half of the demanded speed and after this to linearly decrease to zero, at which point the demanded speed is reached (*remark: S curve*).

d) the *linear first order* mode, in which the rotor speed follows a time varying reference speed with first order linear dynamics and a prescribed time constant, and

e) the *linear second order* mode in which the rotor speed follows a time varying reference speed with second order linear dynamics having prescribed settling time and damping ratio.

Mode (a) is similar (in its end result but not in its means of achievement) to the now well-established *direct torque control* method. Here, the rotor angular acceleration is controlled to follow a demanded angular acceleration with negligible dynamic lag. This mode may be converted to direct torque control by simply dividing the demanded torque by the rotor moment of inertia to obtain the corresponding demanded angular acceleration. It is unnecessary to include the equivalent moment of inertia of the driven mechanical load or other dynamic effects such as friction, since the entire load dynamics may be represented as a time-varying load torque and this is estimated and compensated for in all the control schemes presented.

Modes (b), (c), (d) and (e) are all special cases of mode (a) in which the rotor acceleration is controlled to achieve the desired dynamics. This is true in the sense that the instantaneous demanded rotor shaft angular acceleration is automatically varied using the desired differential equation relating the angular acceleration to the achieved and demanded angular velocities.

It should be noted that the FDC method is a form of *feedback linearisation* for dynamic modes (d) and (e), since the desired closed-loop system obeys a *linear* differential equation. For dynamic modes, such as (b) and (c), however, the desired closed-loop system obeys a *nonlinear* differential equation, and so the more term, *forced dynamic control*, is used to describe the general method.

Mode (b) is a form of soft start control, which is generalised to yield speed changes between any two demanded values at a constant acceleration or deceleration.

Modes (c) and (e) are intended for applications requiring responses to changes of speed that avoid shaking the controlled mechanism. Examples are:

a) a 'drive by wire' electric locomotive in which a sudden change of speed demand from the driver would limit the jerk (i.e., rate of change of acceleration) and
b) a controlled crane in which the rate of build-up of the force in a load being lifted would be limited, in the interests of care in handling the payload.

Mode (d) is particularly useful for applications in which one or more drives are elements of an overall control system, such as variable speed pumps for liquid level control in industrial processes involving inter-connected tanks. The prescribed drive speed control time constants then become adjustable parameters in the design of the overall control system.

In all these modes, the current and magnetic flux vectors producing the torque can be maintained mutually perpendicular as in conventional vector control.

1.1.2 Basic dynamic model of electrical drive

The starting point in creating a drive control system is the differential equation modelling the motor and its driven load. In its basic form, which is applicable to all types of electric motor, this equation relates the electrical torque, $\Gamma_{el}(t)$ developed by the motor and the load torque, $\Gamma_L(t)$, acting on the rotor inertia, to the rotor angular acceleration, $\dot{\omega}_r$:

$$\dot{\omega}_r = \frac{1}{J_r}\left[\Gamma_{el}(t) - \Gamma_L(t)\right]. \tag{1.1.1}$$

For a.c. motors, $\Gamma_{el}(t)$ is a function of the two current components and two magnetic flux components, in either the stator-fixed α, β frame or the rotor-fixed d, q frame, and the particular cases of the induction motor (IM) and permanent magnet synchronous motor (PMSM) will be dealt with in subsequent sections of the book. It will be shown later that it is possible to control the stator voltages so that the real electrical torque follows a demanded value with negligible errors and therefore, for the simple introductory explanation given in this section, $\Gamma_{el}(t)$ will be regarded as the control variable. Fig. 1.1.1 shows a general block diagram of the motor and its mechanical load.

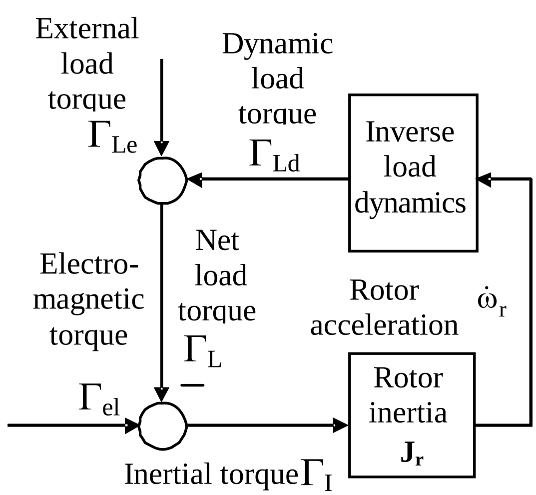


Fig. 1.1.1 Block diagram of motor and its load, together with torque definitions

This diagram is most important in appreciating why the control systems developed in the research programme exhibit the desirable property of *robustness*, i.e., their ability to yield the prescribed dynamic speed responses to their reference inputs despite uncertainties in knowledge of the driven mechanical load and the presence of external load torques. The model will now be explained.

With reference to Fig. 1.2.1, the load torque, Γ_L, can be regarded as composed of two components, the *external load torque*, Γ_{Le}, and the *dynamic load torque*, Γ_{Ld}. Most importantly, the driven mechanical load is represented in the *inverse dynamic form* in the feedback path of the model, so that Γ_{Ld} is the torque component that must be applied to produce a given motion. For example, if a balanced load mass with moment of inertia, J_L, is bolted to the rotor of the electric motor, then $\Gamma_{Ld} = J_L \dot{\omega}_r$. This will be defined as the *inverse dynamic equation* since it yields Γ_{Ld} given the rotor angular acceleration, $\dot{\omega}_r$. Conversely, the dynamic equation of the load mass taken in isolation from the motor is $\dot{\omega}_r = \Gamma_{Ld} / J_L$, since it yields the angular acceleration, given the applied torque. The *inertial torque* is important too. This is the torque applied to the rotor inertia, which yields a given angular acceleration. This is sometimes called the *dynamic torque* but the alternative term, *inertial torque*, Γ_I, is used here to avoid confusion with the *dynamic load torque* previously defined. To illustrate the validity of the model, it is evident from Fig. 1.1.1, that:

$$\Gamma_I = \Gamma_{el} - \Gamma_{Le} - \Gamma_{Ld}, \qquad (1.1.2)$$

but $\Gamma_{Ld} = J_L \dot{\omega}_r$ and $\Gamma_I = J_r \dot{\omega}_r$. Hence:

$$J_r \dot{\omega}_r = \Gamma_{el} - \Gamma_{Le} - J_L \dot{\omega}_r, \qquad (1.1.3)$$

from which

$$\dot{\omega}_r = (\Gamma_{el} - \Gamma_{Le})/(J_r + J_L) . \qquad (1.1.4)$$

This is the expected result.

1.1.3 General equation for forced dynamic control of electrical drive

All that is required is to write down a differential equation for the *desired closed-loop dynamic behaviour*, which relates the rotor angular acceleration, to the rotor angular velocity and the demanded rotor angular velocity, ω_d . This will usually be of the same order as the motor mechanical equation (1.1.1), i.e., first order. In general, this may be written:

$$\dot{\omega}_r = a_d(\omega_r, \omega_d), \tag{1.1.5}$$

where the choice of the *demanded rotor acceleration function*, $a_d(\omega_r, \omega_d)$, determines the *dynamic mode* referred to in section 1.1.1. The motor, as modelled by (1.1.1) is then *forced* to follow the desired *dynamics* of (1.1.5) simply by equating the right hand sides:

$$\frac{1}{J_r}[\Gamma_{el} - \Gamma_L] = a_d(\omega_r, \omega_d). \tag{1.1.6}$$

This will be called the *forced dynamic equation*. It should be noted that in cases where $a_d(\omega_r, \omega_d)$, is a linear function of its arguments, the closed-loop system is linear and equation (1.1.6) will be called the *linearising equation*. The electrical torque required from the motor to achieve the required dynamic mode is then obtained by making Γ_{el} the subject of equation (1.1.6), yielding $\Gamma_{el} = \Gamma_L + J_r a_d(\omega_r, \omega_d)$. In a practical control system, a fairly accurate estimate, $\tilde{J}_r$, of J_r must be used. Fortunately, as shown in the further sections, it is possible to form an estimate, $\hat{\Gamma}_L$ of Γ_L using an observer, and this must be used. As mentioned previously, Γ_{el} is regarded as the control variable, because the motor stator voltages can be controlled to follow a demanded electrical torque, Γ_{el}, with negligible errors. The practicable form of the forced dynamic control law is then:

$$\Gamma_{el\ d} = \hat{\Gamma}_L + J_r \cdot a_d(\omega_r, \omega_d), \tag{1.1.7}$$

It is important to see that the component, $\hat{\Gamma}_L$, of the motor electrical torque in (1.1.7) effectively counteracts the entire load torque, Γ_L , in Fig. 1.1.1, if it is a good estimate and thereby renders the drive control system almost independent of the dynamics of the driven load and the external load torque, by counteracting the dynamic load torque, Γ_{Ld}, as well as Γ_{Le}. The successful operation of the control system therefore depends critically on the load torque observer.

It is possible to force the drive to operate in a dynamic mode of greater than first order and this may be done by means of a digitally integrated model of the desired dynamics. The continuous time form of this model may be written as:

$$\omega_m^{(n)} = f\left(\omega_m^{(n-1)}, \omega_m^{(n-2)}, ..., \omega_m, \omega_d\right), \tag{1.1.8}$$

where ω_m is the model rotor angular velocity and $\omega_m^{(q)} = \dfrac{d^q \omega_m}{dt^q}$, $q = 1, 2, .., n$.

Then the demanded acceleration used in equation (1.1.7) is:

$$a_d = \dot{\omega}_m . \tag{1.1.9}$$

In addition to this, however, a loop must be closed using the estimated rotor angular velocity from the observer, which will be denoted by $\hat{\omega}_r$. This will be achieved by using $\hat{\omega}_r$ to drive the model, so the required continuous-time version of the model becomes:

$$\begin{aligned} \omega_m^{(n)} &= f\left(\omega_m^{(n-1)}, \omega_m^{(n-2)}, ..., \ddot{\omega}_m, \dot{\omega}_m, \hat{\omega}_r, \omega_d\right) \\ a_d &= \dot{\omega}_m \end{aligned} . \tag{1.1.10a}$$

Finally, this dynamic model must be numerically integrated on the digital processor controlling the drive, and this may be done in a straightforward manner using the control canonical state space form and explicit Euler integration. The continuous-time state space model has state variables, $x_1 = \omega_m$, $x_2 = \dot{\omega}_m \dots$ $x_n = \omega_m^{(n-1)}$ and is as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= f\left(x_{n-1}, x_{n-2}, ..., x_3, x_2, \hat{\omega}_r, \omega_d\right) \\ a_d &= x_2 \end{aligned} \tag{1.1.10b}$$

The corresponding discrete time model for digital implementation is then:

$$
\begin{aligned}
&x_1(k+1) = x_1(k) + x_2(k)\cdot h \\
&x_2(k+1) = x_2(k) + x_3(k)\cdot h \\
&\vdots \\
&x_{n-1}(k+1) = x_{n-1}(k) + x_n(k)\cdot h \\
&x_n(k+1) = x_n(k) + \left[f\left(x_{n-1}(k),\ x_{n-2}(k),\ \ldots, x_3(k),\ x_2(k),\ \hat{\omega}_r(k),\ \omega_d\right)\right]\cdot h \\
&a_d = x_2(k+1)\ ,
\end{aligned}
\tag{1.1.11}
$$

where h is the step-length of the numerical integration. One of the *second order linear dynamic modes* applied in this book uses this method.

It is possible in some cases to operate higher order dynamic modes without such a model, if the derivatives of (1.1.10a) are already available as estimates from an appropriate algorithm. Then, the model variable, ω_m, of (1.1.10a) is replaced by the rotor speed estimate, $\hat{\omega}_r$, and $\dot{\hat{\omega}}_r$, which is a_d, is made subject of the equation:

$$
a_d = g\left(\hat{\omega}_r^{(n-1)},\ \hat{\omega}_r^{(n-2)},\ \ldots,\ \dot{\hat{\omega}}_r\ ,\hat{\omega}_r\ ,\ \omega_d\ \right). \tag{1.1.12}
$$

1.2. Demanded Acceleration Functions for Dynamic Modes

The functions, $a_d(\omega_r, \omega_d)$, yielding the first order dynamic modes and other functions yielding the other dynamic modes listed and described in section 1.1.1 will now be given.

1.2.1 Direct acceleration control mode

In this case, a_d , is not a function of ω_r and ω_d , but is an external signal provided by the drive user. The drive will only realise such demanded rotor acceleration, of course, within the limits determined by the maximum electrical torque, the external load torque and the dynamics of the driven mechanical load.

1.2.2 Constant acceleration control mode

In this case, the demanded acceleration is determined by a constant demanded angular velocity, ω_d , and a demanded acceleration time, T_s, and the dynamic torque is then determined by the sign of the angular velocity error:

$$a_d = \frac{\omega_d}{T_s} \operatorname{sgn}\left(\omega_d - \hat{\omega}_r\right), \tag{1.2.1}$$

where $\hat{\omega}_r$ is the filtered rotor speed estimate from the aforementioned observer and $\operatorname{sgn}(x) = \begin{cases} +1 & \text{for } x \rangle 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x \langle 0 \end{cases}$. This form of start-up response is commonly found in industrial electric drives designed to reach a constant demanded speed.

1.2.3 Constant jerk control mode

In this mode, the acceleration during speed-up is increased from zero linearly up to half of the demanded speed and is then decreased to zero linearly at the same rate, at which point the demanded speed is reached. Thus the value of the acceleration derivative 'ε' (i.e., the *jerk*) during speed-up is constant and the maximum acceleration is achieved in the middle of the speed-up interval. The demanded acceleration achieving parabolic movement may be generated as follows:

$$a_d = \begin{cases} \varepsilon t \cdot \mathrm{sgn}(\omega_d - \hat{\omega}_r) & \text{for } t \in (0,\ T_s/2) \\ \varepsilon(T_s - t) \cdot \mathrm{sgn}(\omega_d - \hat{\omega}_r) & \text{for } t \in (T_s/2\ ,\ T_s) \end{cases}\ , \tag{1.2.2}$$

where the constant jerk magnitude is $\varepsilon = 4\omega_d / T_s^2$. It is important to calculate the maximum acceleration magnitude during this dynamic mode, as this determines the maximum motor torque required. This is given by $a_{max} = 2\omega_d / T_s$.

The demanded acceleration of (1.2.2) is only valid for drive starting applications but may be generalised to a second order nonlinear dynamic mode which will continuously control the drive speed from one constant value to another with two intervals of equal and opposite jerk. These intervals are not, in general, of the same duration. This is similar mathematically to the closed-loop form of the time-optimal position controller for a mass moving without friction. The required demanded acceleration equation is derived using the general method presented at the end of section 1.1.3 (ref., equation (1.1.9)):

$$a_d = -a_{max} \cdot \mathrm{sgn}\left(\hat{\omega}_r - \omega_d + \frac{1}{2a_{max}} \dot{\hat{\omega}}_r \left| \dot{\hat{\omega}}_r \right| \right). \tag{1.2.3}$$

As will be seen, $\dot{\hat{\omega}}_r$ is readily available from the observer.

1.2.4 Linear first order control mode

In this case, the closed-loop system becomes a first order linear one with a time constant of T_ω and the demanded acceleration equation yielding this is:

$$a_d = \frac{1}{T_\omega}\left(\omega_d - \hat{\omega}_r\right). \tag{1.2.4}$$

The closed-loop transfer function of the speed control system is then $\dfrac{\omega_r(s)}{\omega_d(s)} = \dfrac{1}{1 + sT_\omega}$. The approximate settling time of this system to reach approximately 95% of the steady-state speed response is well known and given by:

$$T_s = 3T_\omega. \tag{1.2.5}$$

This is the dynamic mode used in the first experiments with the FDC based a.c. drives.

1.2.5. Linear second order control mode

In this case, the desired closed-loop differential equation for the ideal rotor speed is given by:

$$\ddot{\omega}_r = \omega_n^2(\omega_d - \omega_r) - 2\xi\omega_n\dot{\omega}_r\,, \tag{1.2.6}$$

where ω_n is the undamped natural frequency and ζ is the damping ratio. Both of these parameters may be chosen freely. The resulting closed-loop speed control transfer function of the drive is $\frac{\omega_r(s)}{\omega_d(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ and the approximate settling time of this system to reach approximately 95% of the steady-state speed response is given by:

$$T_s = \frac{4.5}{\xi\omega_n}\,. \tag{1.2.7}$$

Following the method presented in section 1.1.3 (ref., equations (1.1.5) to (1.1.8), incl.), a real-time second order model is set up and driven using the speed estimate, $\hat{\omega}_r$, from the observer. The continuous-time version of this model is formed from (1.2.6), and together with the demanded acceleration equation, this is as follows:

$$\begin{aligned} \ddot{\omega}_m &= \omega_n^2(\omega_d - \omega_m) - 2\xi\omega_n\dot{\omega}_m \\ a_d &= \dot{\omega}_m \end{aligned}\,. \tag{1.2.8}$$

To facilitate straightforward numerical integration, the corresponding state-space model, with state variables, $x_1 = \omega_m$ and $x_2 = \dot{\omega}_m$ is as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \omega_n^2(\omega_d - x_1) - 2\xi\omega_n x_2 \\ a_d &= x_2 \end{aligned}\,. \tag{1.2.9}$$

The corresponding discrete time model for digital implementation is then:

$$
\begin{aligned}
x_1(k+1) &= x_1(k) + x_2(k)\cdot h \\
x_2(k+1) &= x_2(k) + \left[\omega_n^2\left(\omega_d - x_1(k)\right) - 2\xi\omega_n x_2(k)\right]\cdot h \,. \\
a_d &= x_2(k+1)
\end{aligned}
\tag{1.2.10}
$$

this particular example, however, a convenient 'short cut' can be taken that avoids the state-space model. Returning to (1.2.6), ω_r is replaced by $\hat{\omega}_r$ and $\dot{\omega}_r$ is replaced by a_d yielding:

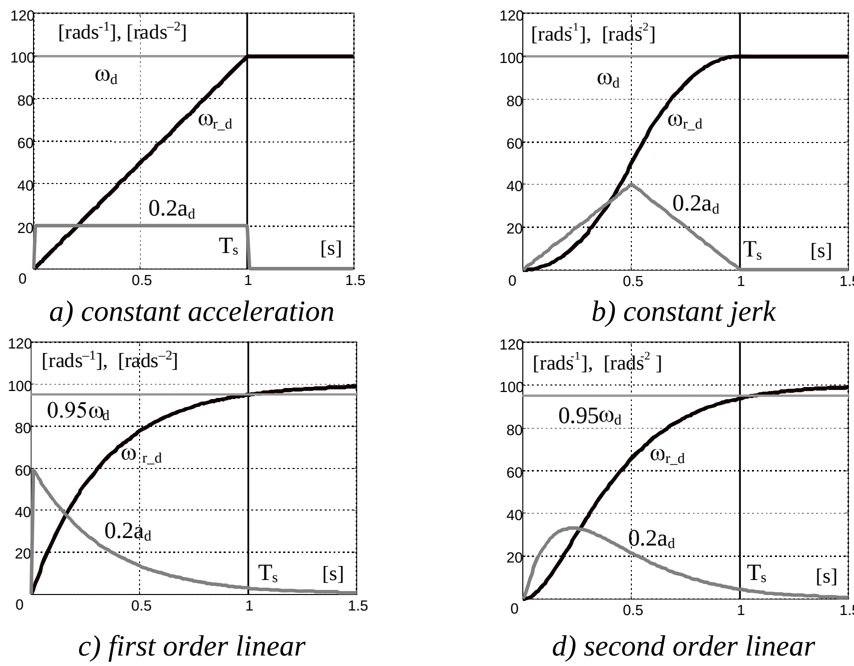
$$
\dot{a}_d = \omega_n^2\left(\omega_d - \hat{\omega}_r\right) - 2\xi\omega_n a_d\,, \tag{1.2.11}
$$

which may be numerically integrated to yield the following iterative algorithm:

$$
a_d(k+1) = a_d(k) + \left[\omega_n^2\left(\omega_d - \hat{\omega}_r\right) - 2\xi\omega_n a_d(k)\right]\cdot h \quad . \tag{1.2.12}
$$

This method has proven successful experimentally as well as that of (1.2.10).

The speed and acceleration profiles for four of the dynamic modes described above (1.2.2 – 1.2.5) are shown in Fig. 1.2.1.



a) constant acceleration *b) constant jerk*

c) first order linear *d) second order linear*

Fig. 1.2.1 Speed and acceleration profiles for individual dynamic modes